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Dynamical study of bare σ pole with $1/N_c$ classifications

Toru Kojo[1] and Daisuke Jido[2]
[1] *RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*
[2] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

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Systematic $1/N_c$ counting of correlators is performed to directly relate quark-gluon dynamics to qualitatively different hadronic states order by order. Both 2q and 4q correlators of σ quanta are analyzed with $1/N_c$ separation of the instanton, glueball, and in particular, two meson scattering states. The *bare* resonance pole with no mixing effects are studied with the QCD sum rules (QSR). The bare mass relation for large N_c mesons, $m_\rho < m_{4q}^{I=J=0} < m_{2q}^{I=J=0}$, is derived. The firm theoretical ground of the QSR at the leading $1/N_c$ analyses is also emphasized.

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Quantum chromodynamical (QCD) descriptions of hadron properties have direct relevance to understanding of the nonperturbative aspects of the strong interaction. For instance, success of the constituent quark model for global hadron spectra has illuminated some properties of chiral symmetry breaking and confinement, and provided the concept of constituent quarks as quasiparticles inside hadrons. Yet, not all hadrons are alike. In addition to the well-established Nambu-Goldstone bosons, there exist some exceptional states, for example, the light scalar mesons (σ, κ, a_0, f_0) [1], newly observed charmonia (X, Y, Z) [2], baryon resonance $\Lambda(1405)$ and flavor exotic $\Theta^+(1540)$ [3]. The studies of these exotic hadrons could provide new viewpoints beyond the simple consitituent quark picture, such as multi-quark components and/or inter-hadron-dynamics.

The lightest scalar meson, σ with $I = J = 0$, is a typical example which has almost all important ingredients as the exotic hadrons. The σ meson includes not only usual $q\bar{q}$ (2q) but hadronic states beyond the constituent quark picture: glueball, $\pi\pi$ molecule, and $qq\bar{q}\bar{q}$ (4q) state with diquark correlation [4]. Thus it is a good laboratory to explore the properties and interplay of these states. Extraction of these properties has direct phenomenological importance to not only the hadron spectroscopy but also nuclear/quark-hadron mater, through the properties of nuclear force [5], chiral order parameter of QCD [6], and diquark correlation [4].

The studies of the σ meson, however, are not straightforward. Since σ can be described as admixture of several hadronic states, it is difficult to identify which hadronic states are responsible to which part of the σ properties. Experimental information is still not satisfactory to derive the definite conclusion. Therefore, as a first step, it is important to theoretically clarify the properties of each hadronic state in the absence of mixing with other states [7] and, at the same time, to illuminate the role of interplay between these hadronic states, by examining the descrepancies between states with no mixing effects and the experimental σ data with all mixing effects.

For this purpose, we introduce the classification based on inverse expansion of number of color, $1/N_c$ [8, 9], for the hadronic states in the correlators made of quark-gluon fields: $\Pi(q^2) = i\int d^4x\, e^{iq\cdot x}\langle T J(x)J^\dagger(0)\rangle$, including contributions from all possible hadronic intermediate states with the same quantum number as $J(x)$. One of the largest virtues of $1/N_c$ expansion is that it directly relates the $1/N_c$ classifications of the quark-gluon graphs to the qualitative classifications of the hadronic states with the same quantum number, in a way that the mixing of these hadronic states are suppressed by higher order quark-gluon graphs of $1/N_c$. Then we can concentrate on the graphs for the hadronic states of our interest, separating mixing effects from higher order of $1/N_c$.

In this letter, we demonstrate this idea in the case of the correlators of 2q and 4q operators with the σ quantum number. On the basis of the $1/N_c$ distinction, we can give an inductive definition of the bare "2q" and "4q" states, being free from the contributions of glueball, instanton, and, in particular, $\pi\pi$ scattering states, which are the origin of the large width ~ 500 MeV [10] and large background in the σ meson spectrum. A novel consequence of our approach is that the bare "4q" state can be investigated independently of 2-meson states, and this explicitly demonstrates the efficiency of the $1/N_c$ distinction of states with the same quantum number but with qualitative differences. The existence and properties of the "4q" state are dynamically studied with comparing them to the "2q" state through the QCD sum rules (QSR) [11], whose theoretical ground is firm in the leading $1/N_c$. We will figure out the importance of the "4q" component with smaller mass than "2q" case by $150 \sim 200$ MeV despite of larger number of quarks participating in the dynamics. This indicates the existence of the nontrivial correlation for the mass reduction of the "4q" system.

The interpolating fields used in this work are summarized as follows: The 2q interpolating fields are described as $J_M^F = \bar{q}\tau_F\Gamma_M q$, where Dirac matrix Γ_M labeled by $M = (S, P, V, A, T)$ for $(1, i\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu})$, respectively, and τ_F ($F = 1, 2, 3$) are the Pauli matrices acting on $q = (u, d)^T$. The 4q operators with the σ quantum number are given (assuming the ideal mixing for the σ meson) by $J_{MM}(x) = \sum_{F=1}^{3} J_M^F(x)J_M^F(x)$ as products of meson operators (Hereafter we take the SU(2) chiral limit for simplicity).

Here we first see the $1/N_c$ linking between quark-gluon dynamics and hadronic states in the case of the 2q correlators. The known facts are as follows: [8, 9]: 1) For quark-gluon diagrams, n-internal quark loops are suppressed by $1/N_c^n$. In terms of hadron, n-meson or multiquark with $(q\bar{q})^n$ production from J_M are suppressed by $1/N_c^n$. 2) The disconnected diagrams with two gluon emission are suppressed by $1/N_c$. In terms of hadron, $q\bar{q}$-glueball mixing is suppressed by $1/N_c$. 3) Instanton effects are suppressed by $\sim e^{-N_c}$. 4) For the meson properties the n-meson couplings are given as $g_{nM} = O(N_c^{(2-n)/2})$. Here 1)∼3) implies that leading diagrams with $O(N_c)$ can be naturally interpreted as the bare "2q" state since the 4q propagation diagrams/$\pi\pi$ scattering, glueball, and instanton contributions do not appear at this order.

Similar identification and subsequent separation are also possible in the case of the 4q correlator, which incorporates the 4q participating diagrams from the beginning. Here we give inductive definition on the "4q" state following the $1/N_c$ based orthogonal conditions: a) "4q" can *not* appear in the leading N_c 2q correlator, b) "4q" can appear in the 4q correlators even after the separation of 2-meson scattering states. These conditions insist that its dynamical origin is different from "2q" and 2-meson molecule states (The glueball and instanton are easily verified to be higher order in $1/N_c$ than those considered below, thus we will not discuss them in the following).

Although this definition gives a convenient starting point to discuss the qualitative difference between hadronic states in the σ meson, the studies of the "4q" component require systematic $1/N_c$ arguments for the 4q correlators, beyond the leading $O(N_c^2)$ quark-gluon diagrams including only 2 planar loops (Fig.1,a), which are naturally interpretted as free 2-meson scattering and are irrelevant for the studies of the "4q" properties. Thus we must proceed to the next leading order of $1/N_c$, $O(N_c)$ diagrams which could include 2-meson scattering, "2q", and "4q" states at this next leading order of $1/N_c$.

The $O(N_c)$ quark-gluon diagrams in the 2-point function could include the various hadronic contributions. The easiest way to classify them is to consider the overlap strength of the operator J_{MM} with hadronic states, involving all the elements of hadronic diagrams. We first classify the overlap strength of the 4q field with the 2 meson states based on $1/N_c$, employing 3-point correlator among the 4q current J_{MM} and two separated meson operators $J_{M'}$ (Fig.1, d-f). An explicit examination of quark-gluon graphs shows that the leading order diagrams are $O(N_c^2)$ for $M = M'$ case, and $O(N_c)$ for $M \neq M'$. Combining these facts with the overlap strength of J'_M with the 2q meson state $|M'\rangle$ is $O(N_c^{1/2})$, the remaining part should be

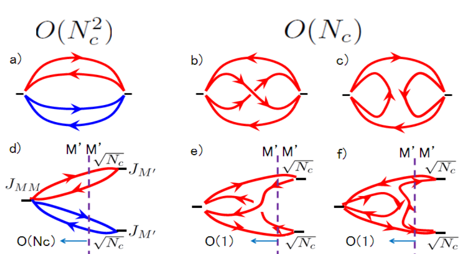

FIG. 1: (Color online) Examples of the $O(N_c^2)$ and $O(N_c)$ quark-gluon diagrams for 2 and 3 point correlators.

$$\langle 0|J_{MM}|M'M'\rangle = O(N_c)\delta_{MM'} + O(1) + ... \quad (1)$$

The first term represents the direct coupling to $|MM\rangle$, while the second term reflects that transition into $|M'M'\rangle$ final state needs the additional interactions. This higher order counting is crucial for the separation of the $\pi\pi$ scattering states from 2-point correlators.

On the other hand, the overlap strength with "2q" and "4q" states cannot be deduced from $1/N_c$ arguments only, and are assumed to be

$$\langle 0|J_{MM}|R\rangle = O(N_c^{1/2}), \quad (R = \text{"2q" or "4q"}) \quad (2)$$

which will be assured later through the dynamical calculations. Similarly, the "4q" state will be identified by examining the quantitative difference of poles in 2q and 4q correlators, which is found to be large enough to distinguish the "4q" and "2q" states. The coupling of R to two mesons is estimated by 3-point function in the same way to obtain the meson couplings performed in Refs. [9] and is found to be $O(N_c^{-1/2})$.

Now we can classify the hadronic states in the 2-point correlators $\langle J_{MM}J_{M'M'}\rangle$ based on $1/N_c$ (See, Fig.1, a-c): (i) If $M = M'$, $O(N_c^2)$ quark-gluon graphs include only the free 2M scattering states in the region $E \geq 2m_M$. Otherwise, the contributions from these quark-gluon diagrams vanish, indicating absence of 2 meson scattering states. (ii) $O(N_c)$ graphs include the 2M or 2M' scattering and possible resonance, "2q" and/or "4q".

Note that the relations (1) in the case of $M, M' \neq P$ nor A indicate that the 2π scattering states are not included up to $O(N_c)$ diagrams, and then the resonance peaks (if exist) *below* $2m_M$ are isolated and have zero width since the decay channel is absent. Therefore, now we can reduce the σ spectrum in the 4q correlator into peak(s) plus continuum *if we retain only diagrams up to* $O(N_c)$.

This separate investigation of the $O(N_c^2)$ and $O(N_c)$ part of QCD dynamics enables to perform the step by step analyses for the 2 meson scattering, "2q", and "4q" spectra. In the application of the QSR, we perform the operator product expansion (OPE) for the correlators in deep Euclidean region ($q^2 = -Q^2$), then translate them, *term by term of* $1/N_c$, into the *integral* of the hadronic spectral function through the dispersion relation:

$$\Pi^{ope}_{N_c^n}(-Q^2) = \int_0^\infty ds\ \frac{1}{\pi}\frac{\mathrm{Im}\Pi^h_{N_c^n}(s)}{s+Q^2} \quad (n = 2, 1). \quad (3)$$

Now we emphasize the practical aspects of $1/N_c$ expansion in the application of the QSR. First, the higher dimension condensates in the OPE, whose values have been not well-known despite of their importance, can

be factorized into the products of known condensates, $\langle\bar{q}q\rangle$, $\langle G^2\rangle$, and $\langle\bar{q}g_s\sigma Gq\rangle$. For example, $\langle(\bar{q}Q)(\bar{Q}q)\rangle = \langle\bar{q}Q\rangle\langle\bar{Q}q\rangle + \sum\langle 0|\bar{q}Q|M\rangle\langle M|\bar{Q}q|0\rangle \to \langle\bar{q}Q\rangle\langle\bar{Q}q\rangle$, holds in leading $1/N_c$ estimation since $\langle\bar{q}q\rangle$ is $O(N_c)$, while $\langle 0|\bar{q}\Gamma q|M\rangle$ is $O(N_c^{1/2})$ [9]. Keeping this merit, we will deduce the final $O(N_c)$ results from the off diagonal correlator $\langle J_{VV}J_{SS}^\dagger\rangle$, whose leading order is $O(N_c)$ thus without the factorization violations at the $O(N_c)$ OPE.

Secondly, the lowest resonance in the reduced $O(N_c)$ spectra $\mathrm{Im}\Pi^h_{N_c}(s)$ for the "2q" and "4q" states, can be described as the sharp peak because of the absence of the decay channel. Applying the usual quark-hadron duality approximations to the higher excited states, $\pi\mathrm{Im}\Pi^h_{N_c}(s) = \lambda^2\delta(s-m_h^2) + \theta(s-s_{th})\pi\mathrm{Im}\Pi^{ope}_{N_c}(s)$, and after the Borel transformation for Eq.(3), we can express the effective mass as

$$m_h^2(M^2; s_{th}) \equiv \frac{\int_0^{s_{th}} ds\; e^{-s/M^2} s\; \mathrm{Im}\Pi^{ope}(s)}{\int_0^{s_{th}} ds\; e^{-s/M^2} \mathrm{Im}\Pi^{ope}(s)}. \qquad (4)$$

s_{th} can be uniquely fixed to satisfy the least sensitivity [16] of the expression (4) against the variation of M, since the physical peak should not depend on the artificial expansion parameter M. This criterion is justified only when the peak is very narrow, and our $1/N_c$ reduction of spectra is essential for its application to allow the QSR framework to determine all physical parameters (m_h, λ, s_{th}) in self-contained way.

In practical application of the QSR, it is essential to reduce errors of finite order trunction of OPE and of the quark-hadron duality approximation. Thus Eq.(4) must be evaluated in the appropriate M^2 window (M^2_{min}, $M^2_{max}(s_{th})$) to achieve the conditions: good OPE convergence for M_{min} (highest dimension terms $\le 10\%$ of whole OPE) and sufficient ground state saturation for M_{max} (pole contribution $\ge 50\%$ of the total) [12, 13, 14]. Without the M^2 constraint, we are often stuck with the *pseudo-peak* artifacts [14] often seen in multiquark SRs. Thus we carry out OPE up to dimension 12 [15] to include the sufficient low energy contributions which is essential to find the reasonable M^2 window [12, 14].

We summarize the numerical values used in the analyses. The gauge coupling constant behaves like $O(N_c^{-1/2})$, and the condensates, $\langle O\rangle = (\langle\bar{q}q\rangle, \langle\alpha_s G^2\rangle, \langle\bar{q}g_s\sigma Gq\rangle)$ are $O(N_c)$. Here we additionaly put simple N_c scaling assumptions: $\alpha_s|_{N_c} = 3\alpha_s/N_c$, $\langle O\rangle|_{N_c} = \langle O\rangle N_c/3$. We use the following values with errors for the $N_c = 3$ case, $\alpha_s(1\mathrm{GeV}) = 0.4$, $\langle\alpha_s G^2/\pi\rangle = (0.33\ \mathrm{GeV})^4$, $\langle\bar{q}q\rangle = -(0.25 \pm 0.03\ \mathrm{GeV})^3$, and $\langle\bar{q}g_s\sigma Gq\rangle/\langle\bar{q}q\rangle = (0.8 \pm 0.1)\ \mathrm{GeV}^2$. The results shown below will be obtained with the central values. We finally show $\langle\bar{q}q\rangle$ and m_0^2 dependence of masses in a wide range since most of errors come from these values.

We start the Borel analyses from the case of the large N_c 2q correlators for the vector meson as a reference and the scalar meson as the "2q" state in the σ meson. The OPE results (up to dimension 6) are nothing but results in the literature with employing the factorization.

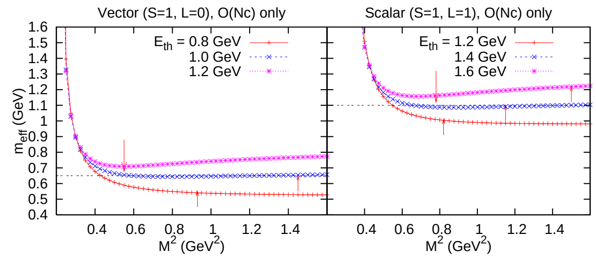

FIG. 2: (Color online) The $O(N_c)$ effective mass plots for vector and scalar mesons in the cases of various E_{th} values. The downward (upward) arrow represent the M^2_{min} ($M^2_{max}(s_{th})$).

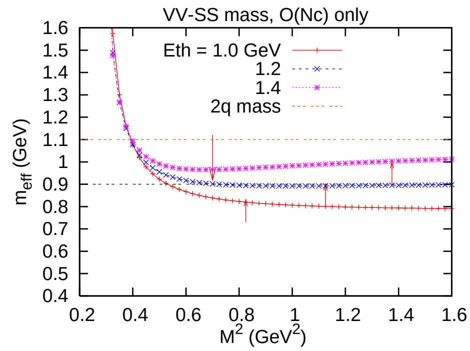

FIG. 3: (Color online) The effective mass plots for $\langle J_{SS}J_{VV}^\dagger\rangle$, including only $O(N_c)$ OPE diagrams. The large N_c "2q" scalar meson mass is also indicated as a reference.

Shown in Fig.2 are the effective mass for the large N_c vector and scalar meson as functions of M^2 for various E_{th}. The downarrow (upperarrows) indicates the value of M^2_{min} ($M^2_{max}(s_{th})$). Following the E_{th} ($\equiv \sqrt{s_{th}}$) fixing criterion, we obtain E_{th} to 1.0 (1.4) GeV for the vector (scalar) meson, and determine the mass as 0.65 (1.10) GeV. The mass splitting $\sim$0.45 GeV between the vector and scalar mesons roughly coincides with the angular excitation energy expected from the naive consitituent quark picture. The reasons for slightly small value of the large N_c ρ meson mass could be due to the absence of the factorization violations [17] if the large N_c scaling of condensates holds. Since our original interests are in the qualitative difference in large N_c mesons rather than their absolute values, thus we will not discuss the details of the absolute values further.

Now we turn to the 4q correlator results of our main interest. We have investigated $O(N_c^2)$ and $O(N_c)$ part of $\langle J_{SS}J_{SS}^\dagger\rangle$, $\langle J_{VV}J_{VV}^\dagger\rangle$, and $O(N_c)$ part of $\langle J_{VV}J_{SS}^\dagger\rangle$. We have checked that the typical effective masses for $O(N_c^2)$ part of $\langle J_{SS}J_{SS}^\dagger\rangle$ ($\langle J_{VV}J_{VV}^\dagger\rangle$) are well above the twice of the large N_c meson mass 2.2 (1.3) GeV, indicating that there is no prominent structure below the free 2-meson threshold, as expected from $1/N_c$ arguments.

The "4q" state can appear from $O(N_c)$ part. Shown in Fig.3 are the effective masses deduced from $\langle J_{VV}J_{SS}^\dagger\rangle$ for E_{th}=1.0, 1.2, and 1.4 GeV. We take the E_{th}=1.2 GeV case and evaluate its mass as $\sim$0.90 GeV, which is obviously lower than that of the "2q" scalar meson case, $\sim$1.10 GeV in large N_c limit, and thus is considered as the mass of the "4q" state. The threshold value

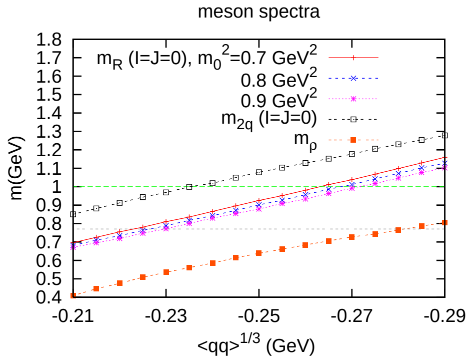

FIG. 4: (Color online) The condensate value dependence of masses of the large N_c mesons, "4q" (m_0^2=0.7, 0.8, 0.9 GeV^2), "2q" scalar and vector states.

1.2 GeV for E_{th}, much below the 2 scalar (vector) meson threshold, 2.2 (1.3) GeV, may be due to the "2q" scalar meson contribution, since our $O(N_c)$ 4q correlators can also include the "2q" contribution. We have also investigated the $O(N_c)$ part of $\langle J_{SS}J^\dagger_{SS}\rangle$ ($\langle J_{VV}J^\dagger_{VV}\rangle$), and obtained the almost same mass 0.80 (0.90) GeV although they could suffer from the factorization violation coming from $O(N_c^2)$ OPE. These 3-independent $O(N_c)$ correlator analyses consistently suggest existence of the "4q" state lighter than the "2q" state.

Finally, we derive a conservative conclusion which does not depend on the details of our numerical parameters, especially $\langle \bar{q}q\rangle$ and m_0^2 (The dependence on the other parameters is relatively small). Shown in Fig.4 are "2q" vector, scalar meson masses, and of the "4q" mass (deduced from $\langle J_{VV}J^\dagger_{SS}\rangle$) as functions of the $\langle \bar{q}q\rangle$ and m_0^2. We found that the inequality $m_\rho < m_{4q}^{I=J=0} < m_{2q}^{I=J=0}$ holds irrespective of details of the condensate values.

The results obtained here suggest that the σ meson has the "4q" component, which is not generated from $\pi\pi$ interactions but from those at the quark-gluon level. Here we comment on Pelaez's elaborated work for the σ meson using the unitarized chiral perturbation with $1/N_c$ expansion [18]. He showed that the σ state as the pole in the $\pi\pi$ scattering amplitudes of T-matrix disappears in large N_c limit in contrast to the case of ordinary mesons such as the ρ meson. This is not contradicted with our results since the $\pi\pi$-"4q" mixing is suppressed by $1/N_c$ and "4q" state is not accessible from the $\pi\pi$ initial states. This is in sharp contrast to the 4q correlator approach including the "4q" state directly generated from 4q current.

We conjecture that the "4q" component may play important role as a building block of the σ meson. To develop this possiblity, we plan to study the 3-point correlator for the "4q"-$\pi\pi$ coupling strength. The coupling strength should be large since the σ in nature is the broad resonance. If this is indeed the case, the σ meson could be described as the 4q core clothed by the $\pi\pi$ clouds. The relative importance of the "4q" state in the σ meson can be investigated through the coupled channel analyses using the effective Lagrangian including not only the π field but also an elementary "4q" field, whose effects are considered to be hidden in the parameters or regulation constants in the usual chiral perturbation approaches. This is somehow related to the recent arguments for the $N^*(1535)$ resonance [19].

$1/N_c$ arguments developed in this work are expected to have the wide applications. First, the spectrum reduction is applicable to the analyses of other tetraquark candidates. We have already obtained results that the effective mass in $I=2, J=0$ channel does not show any stability against M^2 variation, which indicates absence of "4q" states consistently with experiments. Second, the QCD sum rules which are firm in the large N_c limit could provide useful information to models based on the Gauge/Gravity duality (large N_c QCD $\leftrightarrow$ SUGRA), through the properties of the large N_c mesons. All these issues will be reported in future [20].

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